

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch: Diploma (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

a) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \underline{\hspace{2cm}}$

- (A)
- $\log_a e$
- (B)
- $\log_e a$
- (C)
- e
- (D) 1

b) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \underline{\hspace{2cm}}$

- (A) -1 (B) 1 (C) 0 (D) 3

c) $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \underline{\hspace{2cm}}$

- (A) -1 (B) 0 (C) 1 (D) None of these

d) $\frac{d(7)}{dx} = \underline{\hspace{2cm}}$

- (A) 0 (B) 1 (C) -1 (D) 7x

e) $\frac{d\left(\frac{1}{x}\right)}{dx} = \underline{\hspace{2cm}}$

- (A)
- $\frac{1}{x}$
- (B)
- $-\frac{1}{x^2}$
- (C)
- $\log x$
- (D)
- $-\frac{1}{x}$

f) $\frac{d(\sin^{-1} x)}{dx} = \underline{\hspace{2cm}}$

- (A)
- $\frac{-1}{\sqrt{1-x^2}}$
- (B)
- $\frac{1}{\sqrt{1-x^2}}$
- (C)
- $\frac{1}{1+x^2}$
- (D)
- $\frac{-1}{1+x^2}$

If $f(x) = \log \sqrt{x^2 + 1}$ then $f'(0) = \underline{\hspace{2cm}}$.

g) (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 0

h) $\int a^x dx = \underline{\hspace{2cm}}$



(A) $\frac{a^x}{\log_e a} + c$ (B) $a^x \log_e a + c$ (C) $a^x + c$ (D) $\log_e a + c$

i) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \underline{\hspace{2cm}}$

(A) $\cot^{-1} \frac{x}{a} + c$ (B) $\tan^{-1} \frac{x}{a} + c$ (C) $\cos^{-1} \frac{x}{a} + c$ (D) $\sin^{-1} \frac{x}{a} + c$

j) $\int_0^1 x dx = \underline{\hspace{2cm}}$

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) None of these

k) $|(2, 1, 1) + (1, 2, 3)| = \underline{\hspace{2cm}}$.

(A) 10 (B) $\sqrt{43}$ (C) $\sqrt{34}$ (D) None of these

l) $(1, 1, 1) \cdot (-1, 2, -3) = \underline{\hspace{2cm}}$.

(A) -2 (B) 2 (C) 1 (D) -1

m) $(2, -1, 3) \times (-4, 2, -6) = \underline{\hspace{2cm}}$.

(A) $(-8, -2, -18)$ (B) $(8, 2, 18)$ (C) $(0, 0, 0)$ (D) None of these

n) If the two vectors \vec{a} and \vec{b} are perpendicular (orthogonal) then $\vec{a} \cdot \vec{b} = \underline{\hspace{2cm}}$.

(A) 0 (B) 1 (C) -1 (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

a) If $\vec{a} = 3i - j - 4k$, $\vec{b} = -2i + 4j - 3k$ and $\vec{c} = -i + 2j - 5k$ then find $|\vec{a} + 2\vec{b} - \vec{c}|$.

(5)

b) Find unit vector which is perpendicular to $\vec{a} = 5i + 7j - 2k$ and $\vec{b} = j - 2k + 3i$.

(5)

c) Evaluate: $\lim_{\theta \rightarrow 0} \frac{\cos \theta - \cot \theta}{\theta}$

(4)

Q-3

Attempt all questions

(14)

a) Prove that $\lim_{x \rightarrow \infty} \left[\sqrt{x} (\sqrt{x+p} - \sqrt{x}) \right] = \frac{p}{2}$.

(5)

b) Evaluate: $\lim_{x \rightarrow (-2)} \frac{x^3 + 2x^2 + x + 2}{x^2 + x - 2}$

(5)

c) Find $\frac{dy}{dx}$ if $y = \frac{1 + \sin x}{1 - \sin x}$.

(4)

Q-4

Attempt all questions

(14)

a) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$.
(i) Find velocity at $t = 0$. (ii) Find acceleration at $t = 1$.

(5)

b) Find $\frac{dy}{dx}$ if $y = \log \left[x + \sqrt{x^2 + a^2} \right]$.

(5)

c) If $\vec{a} = (2, -3, -1)$ and $\vec{b} = (1, 4, -3)$ then find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$.

(4)



Q-5

Attempt all questions

(14)

- a) Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$. **(5)**
- b) Find $\frac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. **(5)**
- c) Evaluate: $\lim_{n \rightarrow \infty} 4 \left[\frac{1^3 + 2^3 + 3^3 + \dots + n^3}{n^4} \right]$ **(4)**

Q-6

Attempt all questions

(14)

- a) Constant forces $3i - j + 2k$ and $i + 3j - k$ act on a particle and the particle moves from the point $2i + 3j + k$ to the point $5i + 2j + 3k$. Find the work done by the forces. **(5)**
- b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \cos ecx} dx = \frac{\pi}{4}$. **(5)**
- c) Find derivative of $y = \sqrt{x}$ using first principle. **(4)**

Q-7

Attempt all questions

(14)

- a) Evaluate: $\int x e^x dx$ **(5)**
- b) Find the area of the region bounded between curve $y^2 = 4x$ and line $x = 2$. **(5)**
- c) For what value of p , vectors $2i + 3j - k$ and $pi - j + 3k$ are perpendicular to each other? **(4)**

Q-8

Attempt all questions

(14)

- a) If $\frac{dy}{dx} = 4x^2 + 6x - 1$ and $y = 5$ when $x = 2$, represent y as a function of x . **(5)**
- b) Find $\frac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. **(5)**
- c) Evaluate: $\int x \sqrt{x^2 - a^2} dx$ **(4)**

